Exercise 48

Find the horizontal and vertical asymptotes of each curve. If you have a graphing device, check your work by graphing the curve and estimating the asymptotes.

$$y = \frac{2x^2+1}{3x^2+2x-1}$$

Solution

Calculate the limits as $x \to \pm\infty$ to determine the horizontal asymptote. In the second limit, make the substitution, $x = -u$, so that as $x \to -\infty$, $u \to \infty$.

$$\lim_{x\to\infty} \frac{2x^2+1}{3x^2+2x-1} = \lim_{x\to\infty} \frac{2+\frac{1}{x^2}}{3+\frac{2}{x}-\frac{1}{x^2}} = \frac{2+0}{3+0-0} = \frac{2}{3}$$

$$\begin{aligned}\lim_{x\to-\infty} \frac{2x^2+1}{3x^2+2x-1} &= \lim_{u\to\infty} \frac{2(-u)^2+1}{3(-u)^2+2(-u)-1} \\ &= \lim_{u\to\infty} \frac{2u^2+1}{3u^2-2u-1} \\ &= \lim_{u\to\infty} \frac{2+\frac{1}{u^2}}{3-\frac{2}{u}-\frac{1}{u^2}} \\ &= \frac{2+0}{3-0-0} \\ &= \frac{2}{3}\end{aligned}$$

Therefore, the horizontal asymptote is $y = 2/3$. The vertical asymptotes are found by setting what's in the denominator equal to zero and solving for x.

$$3x^2+2x-1=0$$

$$(3x-1)(x+1)=0$$

$$x = \frac{1}{3} \quad \text{or} \quad x = -1$$

The function is graphed versus x below with the asymptotes labelled.